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INSTABILITIES AND GROWING WAVES IN ELECTRON BEAM DEVICES

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Abstract—Some forms of electron flow are unequivocally unstable in the sense that perturbations grow with time. Excessive currents between grids or through tubes in the presence or absence of neutralizing positive charge provide examples. Such instabilities are not associated with wave components which grow with distance for real frequencies. Thin beams in magnetic fields also show instability under the action of space charge forces. Tubes sometimes exhibit gain in the absence of any wave component which exhibits spatial growth for real frequencies.

There is a large class of amplifiers whose operation can be explained in terms of spatially growing waves. The physical mechanism can be variously described as instability in a moving co-ordinate system, as a result of a negative dielectric constant at the frequency of operation which causes electrons to attract rather than to repel one another, as a result of coupling between positive energy and negative energy waves, and in other terms.

In the easitron, an inductive wall results in growing waves. In the resistive wall amplifier, the abstraction of power from negative energy waves results in growth. In the travelling-wave tube amplifier, a positive power circuit wave is coupled to a negative power space charge wave. In the double stream amplifier, the negative power space charge wave of the faster stream is coupled to the positive power space charge wave of the slower stream. In velocity jump and rippled stream amplifiers, periodic discontinuities couple the positive power and negative power waves of the same stream. In wave-type parametric amplifiers, moving periodic discontinuities couple two unattenuated waves.

The explanations listed are convenient in connexion with the particular device, rather than unique.

SUPPOSE that, as shown in Fig. 1, we inject an electron current of density J_0, composed of electrons with a velocity u_0 specified by a potential V_0, into the space between two parallel short-circuited grids. Suppose we gradually increase the current density. The potential in the space between the grids will gradually fall and, then, as a limiting current density is reached, it will drop abruptly to zero, causing the return of some electrons. The total current density which can pass between the grids decreases as the distance L between the grids is increased. The current density is (FAY, SAMUEL and SHOCKLEY, 1939):

$$J_0 = 18{\cdot}6 \times 16^{-6} V_0^{3/2}/L^2. \tag{1}$$

Here the units are amperes per square centimetre, volts and centimetres.

The mechanism seems clear. An increase in charge density decreases the potential, and a decrease in potential increases the charge density. Above some charge density, the process runs away. One can in fact apply the equations of LLEWELLYN and PETERSON (1944) to determine the stability of disturbances and one indeed finds that for currents just above the limiting value there is an exponentially increasing disturbance (PIERCE, 1944a).

One might think that the presence of a fixed positive charge just neutralizing the average electron charge ('space-charge neutralization') would avoid such an instability, but it merely increases the limiting stable current density (PIERCE, 1944b) to

$$J_0 = 104 \times 10^{-6} V_0^{3/2}/L^2. \tag{2}$$

A limiting stable current (not current density), I_0, can also be found for a long space-charge neutralized beam, (PIERCE, 1944b),

$$I_0 = 190 \times 10^{-6} V_0^{3/2} \tag{3}$$

but experiment indicates that the attainable current actually corresponds very closely to that calculated in the absence of ions,

$$I_0 = 29{\cdot}3 \times 10^{-6} V_0^{3/2}. \tag{4}$$

When the positive charge consists of real ions which can move, oscillations can occur (PIERCE, 1948) at

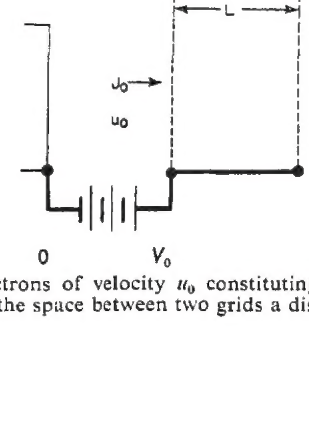

FIG. 1.—Electrons of velocity u_0 constituting a current J_0 injected into the space between two grids a distance L apart.

currents lower than that given by (3), these may reduce the ion density. An alternative to the constant potential assumed in deriving (3) is a sequence of virtual cathodes (PIERCE, 1944b).

It is interesting to note that the limiting number density of electrons corresponding to (3) or (4) can be obtained by dividing the current I_0 by the electron velocity u_0 times the electron charge e times the area of the tube A. This leads to

$$n = 2{\cdot}00 \times 10^7 \; V_0/A \tag{3a}$$

$$n = 3{\cdot}08 \times 10^6 \; V_0/A. \tag{4a}$$

What the implications of this may be for electrons or ions with a velocity distribution moving in the tube of a fusion machine I would not care to guess. We may note that the number densities will be the same in the case of singly charged ions.

This instability in the presence of ions is interesting in that the space-charge waves in a neutralized moving beam of electrons are unattenuated for real frequencies. Thus, one need not have spatially growing waves at real frequencies or temporally growing waves for real phase constants in order to have temporally growing disturbances in the overall system. The klystron amplifier demonstrates that one does not need spatially growing waves at real frequencies in order to amplify a signal.

The instability of the diode at high currents shows the importance of boundary conditions. We can transform into any set of moving co-ordinates we wish in discussing the waves in electron flow, but the stability of such flow depends strikingly on the speed of the electrons with respect to the grids and the distance between the grids.

On the other hand, spatially 'increasing' solutions at real frequencies do not always lead to gain. To see this we need merely note exponentially 'spatially growing' solutions in waveguides at frequencies below cut-off frequency.

One way to avoid drawing incorrect conclusions from a superficial examination of particular wave-type solutions is to solve the overall problem posed by the physical system. This can be done by fitting the boundary conditions by means of a collection of special solutions or waves (PIERCE, 1944b), or it can be done by a transform method (WALKER, 1953).

It has been noted that in some cases an increasing oscillation (WALKER, 1953) or a spatial gain in power (in the klystron) exists in the absence of any waves which increase spatially at real frequencies. In a number of electron beam devices, however, both theory and experiment indicate that the amount of gain is very closely related to the increase along the length of the device of an increasing wave component of the overall disturbance.

Sometimes this increasing wave is of a rather special type. Thus, a thin hollow cylindrical beam of electrons in a longitudinal magnetic field exhibits growing waves at frequencies below a limiting frequency and the growth of these disturbances can be explained by a very elementary consideration of the forces acting on line charges parallel to a uniform magnetic field (PIERCE, 1956).

Another special type of growth is exhibited in the 'easitron,' in which an electron beam is surrounded by a sequence of resonators which are not coupled to one another. Growing waves occur at frequencies such that the impedance of the resonators is inductive. We can explain this by saying that in ordinary space the dielectric constant is positive and electrons repel one another. For electrons surrounded by resonators, at certain frequencies bunches of electrons attract one another and the bunching of the electron stream increases as the electrons travel along. A cloud of ions can serve as a 'resonator' in the absence or presence of a longitudinal magnetic field (PIERCE, 1948).

Indeed, we can note that in the travelling-wave tube and related devices the electrons see an inductive impedance in a growing wave. It is, however, convenient to think of the growing waves in travelling-wave tubes and related devices in somewhat different terms.

The growing waves which account for the gain of many electron beam devices can be explained in terms of a negative-power space-charge or plasma wave on a moving stream of electrons. Such a stream characteristically exhibits a pair of space-charge waves. One is a *slow wave* whose phase velocity is less than the electron velocity. The other is a *fast wave* whose phase velocity is greater than the electron velocity. The slow wave has negative power because in setting it up the electrons have to be bunched together in regions of less than average velocity. In the fast wave the electrons are bunched together in regions of greater than average velocity, and the power is positive. For both waves the group velocity is in the direction of electron motion.

When an electron beam is surrounded by an electrically lossy tube, the slow wave continually supplies energy to the lossy tube and hence it grows in amplitude. This *resistive wall amplifier* is perhaps the most direct exhibition of the negative power of the slow space-charge wave.

Growing waves are also encountered when an electron stream is shot through a cloud of ions (PIERCE, 1948), near a second electron stream (HAEFF, 1948;

PIERCE and HEBENSTREIT, 1949), or near a circuit which supports an electromagnetic wave (KOMPFNER, 1946).

In the double-stream amplifier the increasing wave results from coupling between the slow negative-power wave of the faster stream and the fast, positive-power wave of the slower stream. An increasing and a decreasing wave result; both have the same phase velocity. For each, the sum of the negative power of the slow-wave component and the positive power of the fast-wave component is zero so that the overall increasing or decreasing disturbance has no power.

In the travelling-wave tube the slow, negative-power space-charge wave is coupled to an electromagnetic wave which has positive power. Again, the growing and decreasing waves have no power. At the output end the electromagnetic wave is abstracted as output. The emerging electron stream supports a large negative-power space-charge wave, so that its kinetic power is less than the kinetic power of the undisturbed electron flow.

The slow space-charge wave of an electron stream can be coupled to the fast space-charge wave of the same stream by periodic variations of the potential or diameter of the structure surrounding the beam, or by periodic variations of the beam diameter (FIELD, TIEN and WATKINS, 1951; TIEN and FIELD, 1954; BIRDSALL, 1954). This also results in zero-power increasing and decreasing waves.

It is of interest to consider some aspects of these growing waves. It has sometimes been implied that growing waves grow because energy is added to them along their length. The growing waves we have considered have zero energy. Power can be derived from them by abstracting or destroying part or all of a positive-energy component of the wave, as the electromagnetic circuit component of the growing wave in a travelling-wave tube.

It is also interesting to note that the growing waves in double-stream amplifiers, rippled-stream or velocity jump amplifiers and travelling-wave tubes are made up of unattenuated component waves having group velocities in the same direction. Thus each component wave can be set up at the cathode-end of the electron stream, and when the components are coupled together they will interact so that the disturbance grows in the direction of electron motion.

In the backward-wave amplifier or oscillator (WALKER, 1953) the slow space-charge wave of the electron stream interacts with an electromagnetic wave having a group velocity opposite to the electron flow; the interaction is attained because the wave has a field component with a phase velocity in the direction of electron flow. Both interacting components have negative power in the direction of electron flow and hence they couple to give unattenuated waves. Thus, the gain of the backward-wave amplifier is not due to an increasing wave. The gain occurs because the a.c. circuit voltages of the two unattenuated waves involved add at the output (cathode) end and nearly cancel at the input (collector) end of the tube. In the backward-wave oscillator, the circuit voltages exactly cancel at the collector end of the circuit.

We have seen that in the rippled-beam or velocity-jump tube, periodic features of the beam or of its environment can result in gain. A *moving* feature which partially reflects a wave can transfer energy to the wave (PIERCE, 1948). The reflecting feature could be an electric field, a space-charge or plasma wave, a sound wave or a cloud of particles. A sequence of such reflecting features can be so spaced that their effects add in phase (PIERCE, 1959). Amplification of waves caused by such a periodic sequence of moving reflectors is an example of *parametric amplification*.

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